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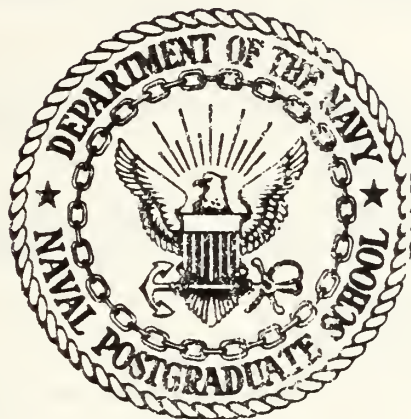






# NAVAL POSTGRADUATE SCHOOL

Monterey, California



## THESIS

COMPARISON OF WAVE CELERITY  
THEORIES WITH FIELD DATA

by

Michael R. Syvertsen

March 1983

Thesis Advisor:

E. B. Thornton

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Comparison of Wave Celerity Theories with Field Data

by

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Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN METEOROLOGY AND OCEANOGRAPHY

from the

NAVAL POSTGRADUATE SCHOOL  
March 1983



## ABSTRACT

Three independent wave celerity data sets, measured on natural beaches, are compared with linear, bore, solitary, and hyperbolic wave theories. In the range of relative water depths ( $.006 < h/T^2 < 13 \text{ cm/s}^2$ ) and wave heights ( $.1 < H/T^2 < 3 \text{ cm/s}^2$ ) tested, hyperbolic wave theory, which is an asymptotic form of cnoidal theory in shallow water, agreed most closely with measured wave celerities. Linear wave theory also gave satisfactory results; but bore and solitary wave theories overestimated the observed wave speeds. It is concluded that the observed waves are weakly dispersive in amplitude and that care must be taken to apply the theories only in their regime of validity.



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## I. INTRODUCTION

Considering all the various theories available to describe wave celerity, surprisingly little work has been oriented toward comparing wave theories with data, particularly field data. Wave celerities acquired in the laboratory were tested against wave theories by LeMehaute et al (1968); they concluded that cnoidal wave theory as proposed by Keulegan and Patterson (1940), with equations and tables by Masch and Wiegel (1961), gave a 'best fit' solution. Other laboratory studies have suggested appropriate wave celerities to be described by solitary wave theory (Ippen and Kulin, 1955) and (Kishi and Saeki, 1967), Stokes theory (De, 1955), hyperbolic/cnoidal theory (Iwagaki, 1968). Celerities measured in the field have been compared with linear theory (Thornton and Guza, 1982), and bore theory (Bradshaw, 1982) and (Suhayda and Pettigrew, 1977). The validity of various theories depends upon the relative depth ( $h/L$ ), where  $h$  is the water depth and  $L$  is the wavelength, and the relative waveheight ( $H$ ) measured in terms of a wavelength ( $H/L$ ) or depth ( $H/h$ ).

In this study, various celerity formulae given by progressive wave theories are tested against celerities



measured in the field. To test the theories against data, the phase speed equations require some combination of water depth, waveheight, and wave period ( $T$ ). Several relatively large field data sets did not provide one or the other of these parameters and could not be used. Data which represents reflected waves from a steep beach such as a data set collected at Fort Ord Beach, California (Sallenger et al, 1983) could not be used. Also, another large data set, collected by University of California, Berkeley, California (Moffitt, 1953) using photographic methods, was not included in the study. This is because the waveheight was omitted in the measurements. Waveheight must be included to test higher order theories, especially if amplitude dispersion is to be examined.

Five wave theories were compared with wave celerities measured in the field in water depths from .07 to 10.0 meters, for wave heights from .12 to 1.0 meter, and for periods varying between 4.7 and 18.3 seconds. Stokes (third order) theory as presented by Hunt (1953) proved unsuitable due to the relatively shallow water depths encountered. The four remaining theories tested are: linear theory; solitary theory as modified by Laitone (1959); bore theory as given



by Keller et al (1960), and hyperbolic theory of Iwagaki (1968), which is an asymptotic form for cnoidal theory in shallow water.

The data have been plotted as a function of waveheight and water depth versus period in Figure 1. This chart, adapted from LeMehaute (1976), attempts to quantify regimes of applicability. For example, the diagram indicates that the data does not fall in the Stokes (third order) theory regime where experimentation proved to be true. Additionally, the Ursell (Ur) parameter ( $Ur = HL^2/h^3$ ), plotted as a dashed line, is commonly used to parameterize the nonlinear waves in shallow water (Thornton and Guza, 1982). For Stokes theory to be valid, the Ursell parameter should be small, which is not the case for most of the data considered. In the case of very long waves in shallow water, the Ursell parameter becomes meaningless since it is directly proportional to the square of the wavelength.

To determine the range of validity for various theories, the underlying assumptions are examined. All the theories presented assume: the motion to be irrotational, the fluid to be incompressible, no mean current flow, and normal wave incidence. As a consequence of the wave theory assumptions,



the Ursell parameter can now be quantified and is included for comparison purposes. The assumptions for the various theories and their celerity equations are as follows:

LINEAR:

$$c = (gk \tanh kh)^{1/2} \quad (1)$$

Assumes:  $H/h \ll 1$ ,  $H/L \ll 1$ ,  $h/L \ll 1$ , and  $Ur \gg 1$  for shallow water,  $h/L \gg 1$  and  $Ur \ll 1$  for deep water.

SOLITARY:

$$c = \sqrt{gh} \left[ 1.0 + \frac{1}{2} \left( \frac{H}{h} \right) - \frac{3}{20} \left( \frac{H}{h} \right)^2 \right] \quad (2)$$

Assumes:  $H/h < 1$ ,  $h/L < .1$ ,  $Ur=0$  (1).

BORE:

$$c = \sqrt{gh} \left( 1.0 + \frac{H}{h} \right)^{1/2} \left( 1.0 + \frac{H}{2h} \right)^{1/2} \quad (3)$$

Assumes:  $H/h=O(1)$ ,  $h/L \ll 1$ ,  $Ur \gg 1$  (very shallow water).



# HYPEREOLIC:

$$c = \sqrt{gh} \left(1.0 - \frac{H}{2Kh}\right) \left[1.0 + \left(1.0 + \frac{H}{Kh}\right) \frac{H}{h} \left(\frac{1}{2} - \frac{1}{K}\right) + \right. \\ \left. \left(1.0 + \frac{2H}{Kh}\right) \left(\frac{H}{h}\right)^2 \left\{ \frac{1}{K} \left(\frac{1}{K} - \frac{1}{4}\right) - \frac{3}{20} \right\} \right] \quad (4)$$

Assumes:  $H/h < 1$ ,  $h/L < .1$ ,  $Ur \gg 1$ , where  $K$  is defined below. Iwagaki (1968) linearized the computationally difficult Jacobian elliptic function into a modulus  $K$ , where:

$$K = \frac{T\sqrt{g/h} \sqrt{3}}{4} \left(\frac{H}{h}\right)^{1/2} \left[1.0 - a \left(\frac{H}{h}\right)^n\right]^m \quad (5)$$

in which  $a=1.3$ ,  $n=2$  and  $m=.5$  for  $H/h \leq 0.55$

and  $a=0.54$ ,  $n=1.5$  and  $m=1$  for  $H/h > 0.55$

This approximation greatly simplified the analysis.



## II. EXPERIMENTS

Three field data sets are used to compare with various theories. Experiments conducted at Torrey Pines Beach and Leadbetter Beach, California were both part of the Nearshore Sediment Transport Study (NSTS) (Seymour and Duane, 1978). These beaches were chosen for their relatively simple beach plan, both unbarred, and essentially straight and parallel nearshore contours. The third data set was from Seven Mile Beach, Australia.

### A. SEVEN MILE BEACH, AUSTRALIA

The Seven Mile Beach (Shoalhaven Bight) experiment on the south coast of New South Wales, Australia, was conducted in early 1982 by Bradshaw (1982). He examined the relationship between bore velocity, height, and water depth by analyzing movie camera pictures using stakes driven into the sand as references. This technique examines each individual wave by counting the number of movie frames between the four reference stakes to determine wave speed while reading bore height and water depth directly from the incrementally marked stakes.



The beach is composed of fine quartz sand and has multiple offshore bars. The beach slope outside was 0.04, while, the inner surf zone had a slope of 0.03. The waves generally broke on the outer bars and then reformed and propagated as bores inside the surf zone. The outside breakers ranged from 1.0 to 1.5 meters.

The data are limited to only shallow water bores inside the breaker line, where bore heights were from 0.14 to 0.30 meters in water depths of 0.07 to 0.42 meters. Data points are plotted as circles in Figure 1 and generally reside in the very shallow water, small waveheight regime. Of the 27 data points available, three points were discarded because bore height greatly exceeded the water depth ( $H/h > 3$ ). Exact period/frequency data was not available for the 27 runs; however, during the two-day experiment, an 8 to 12 second period was observed. Hence the mean (10 second) period is chosen for computational purposes. This assumption is adequate since only shallow water bores were considered and they are essentially nondispersive.

#### B. TORREY PINES BEACH, CALIFORNIA

Field measurements were made at Torrey Pines Beach near San Diego, California in August 1978. A detailed



description of the experiment can be found in Guza and Thornton (1980), Guza and Thornton (1982), and Thornton and Guza (1982). Torrey Pines is a gentle sloping (.02) beach and is composed of fine grain sand. The waves were generally narrow banded and approached the shore at a near normal angle. Directional properties of the waves were measured using a linear, five pressure sensor array in 10 meters depth. The angle of swell approach was limited to the maximum and minimum of  $\pm 15$  degrees due to sheltering by offshore islands and coastline restrictions, but the angle was generally less than  $\pm 5$  degrees. Refraction analysis showed that the predominant swell waves ( $T = 13$  sec.) starting at 15 degrees in 10 meters depth results in angles of incidence of 8.5 degrees in 3 meters depth and 4.9 degrees in 1 meter depth (Thornton and Guza, 1982).

The 92 data points, plotted as +'s in Figure 1, are generally restricted to the shallow water regime. Waveheights ranged from small to 2.0 meters, water depths at sensor locations from 0.27 to 7.0 meters, and mean periods from 9.0 to 18.0 seconds. The experimental domain included spilling or mixed spilling and plunging breakers.



### C. LEADBETTER BEACH, CALIFORNIA

The Leadbetter Beach, Santa Barbara, California, experiment was conducted during the period of 30 January to 23 February 1980. Experimental details are described in Gable (1981). Leadbetter is a relatively straight, steeper sloping (.05) beach composed of fine to medium, well-sorted sand. A series of storms, resulted in abnormally large waves and beach erosion for the period commencing 4 February, 1980. The storms formed what has been described as a '50 year' storm event. Measured wave heights during the experiment ranged from 0.18 to 1.9 meters, with sensor depths up to 10 meters, and periods from 4.7 to 16 seconds. Breaking wave types were of both the spilling and plunging variety. The 83 Santa Barbara data points, plotted as triangles in Figure 1, exhibit the largest waveheights and water depths examined.

Leadbetter Beach has an east-west orientation which is counter to the north-south orientation of the California coastline. The predominant northwest ocean swell entering the narrow gap between Point Conception and the Channel Islands would have to refract nearly 90 degrees to approach normal to the beach. As a consequence, the ocean swell



waves approach at a relatively large, well focused angle from the west. At other times, storm waves generated inside the Channel Islands approached at large angles from the east. Therefore, the wave angularity has to be considered in the wave celerity calculations.

Incident offshore sea-swell was measured using a four pressure sensor square array with 6 meter legs located in a water depth of 8 meters. From these sensors, the mean incidence angle at the peak frequency was determined for the nine data sets. The wave rays were then manually refracted, using Snell's Law, from the pressure array location to a point where the bottom contours could be considered straight and parallel to the beach. Shoreward of this point, the incident wave angles were calculated using a constant refractive coefficient. Deep water angles varied between  $\pm 20$  degrees and were refracted into 4 meters of water from there into the shallower water region of the current sensors. For example, wave angles of 20 degrees in 8 meters depth, period of 12.0 seconds, resulted in refracted angles of 10 degrees in 3 meters decreasing to 5 degrees in 1 meter of water.



Spectral methods were utilized to determine mean celerities and wave heights using current and pressure sensor data from San Diego and Santa Barbara. The celerity and wave height calculations are an average over many waves. Since the record lengths were 34 minutes and the mean period (T) was about 12 seconds, approximately 170 waves are averaged for the spectral estimate of wave celerity. This is in contrast to the camera methods of Bradshaw (1982) where each individual wave was photographed as it passed the reference stakes and waveheight and water depth were read directly.

Pressure and current meter data were telemetered to shore where they were digitally recorded at a rate of 64 samples/second. The data were averaged to 2 samples/second which results in a Nyquist frequency of 1.0 Hertz. Records were then compiled into 4096 data points. By breaking the series into 256 point records, the phase, kinetic energy, and coherency spectra were computed with 32 degrees of freedom.

Celerity spectra were calculated from the phase spectra measured between adjacent pairs of current meters located in a line normal to the beach. The actual celerity ( $C_x$ ) was computed using:

$$C_x(f) = \frac{2\pi f \Delta x}{\phi(f)} \quad (6)$$



where,  $(\Delta x)$  is the distance between sensors,  $(\phi)$  is the phase difference between sensors, and  $(f)$  is the frequency.

It was shown by Thornton and Guza (1982), for the Torrey Pines data, that the waves can be considered frequency non-dispersive such that the celerity spectra is constant, at least across the energetic region of the spectrum. Hence, a mean celerity, representative of the entire spectrum, was chosen at that value corresponding to the peak frequency in the energy spectrum and is used in the data comparison here. Also, the frequency at the spectral peak generally coincided with the maximum coherency.

Hence, the Torrey Pines Beach celerity data were calculated using (6), where  $f = f(\text{peak})$ . At Santa Barbara the wave angularity was taken into account. Since the celerity spectrum was calculated using instruments in a line normal to the beach, only the x component of celerity is measured directly. The total 'mean' phase speed is calculated using:

$$C = C(f_p) = \frac{C_x(f_p)}{\cos \hat{\alpha}(f_p)} \quad (7)$$

where,  $\hat{\alpha}(f_p)$  is the mean angle of wave incidence at the peak frequency  $(f_p)$  - calculated as explained above.



The root mean square of waveheights was calculated from the sea surface variance  $M_0$ , where:

$$H_{RMS} \approx \sqrt{8 M_0} \quad (8)$$

The sea surface elevation variance is proportional to the potential energy. It is assumed that the potential and kinetic energies are equal in the wave system. Hence,  $M_0$  was calculated using the current meter kinetic energy or using the potential energy of the pressure sensors by applying the appropriate linear theory spectral transfer function. It is emphasised that when utilizing spectral decomposition of the current sensors, the data was averaged in time such that the waveheights are representative over the entire time of the measurement.

Mean water depths were measured from pressure sensors associated with the current meters.

The error associated with Torrey Pines Beach celerity calculations is estimated to be within 5% for the deep water sensors and decreases to 1.2% shoreward (Thornton and Guza, 1982). The error estimates are based on the stability of the phase estimates and measurement errors. Estimating sensor spacing errors, depth errors, and refraction problems account for the larger error value in deeper water.



Additional errors can be attributed to the modification of the phase speed due to mean currents. The mean currents were not subtracted out during spectra computations, hence x (off shore) and y (longshore) currents must be addressed. Leadbetter Beach, with the largest angles of swell incidence, exhibited a nominal longshore component of 40 cm/sec. (max of 80 cm/sec) in 1 meter of water, decreasing seaward, to 8 cm/sec. in 3 meters of water. Since maximum incidence angles were on the order of 10 degrees in 3 meters and 5 degrees in 1 meter, the error associated with the measured celerity is on the order of 0.5% (1.0% maximum) and hence it is negligible. The current meters were located in the lower half of the water column. The measured mean currents generally exhibited a slight net offshore flow which is presumed to be balanced by the onshore mass transport in the crest-trough region. Therefore, the net on-offshore mass transport is expected to be significantly small and its effect on the celerity can be considered negligible (Thornton and Guza, 1982).



### III. RESULTS

The three data sets are compared in Figure 1 showing the approximate regions of validity for the various theories. The data do not compare well with the breaking criterion for solitary waves of  $H/h = 0.78$ . Note that approximately 15 data points fall outside the  $H/h=0.78$  curve. These particular waves were part of a study looking at bores during their run-up excursion (Bradshaw, 1982). The waves have exceeded the maximum limit for a solitary wave, but bores do not necessarily compare with solitary wave theory. Nevertheless, studies have indicated that bores should be classified according to their height-to-depth ratio ( $H/h$ ). Peregrine (1966) places bores into three categories. For  $H/h < 0.28$ , the bore is called undular and is composed of a series of undulations radiating behind the leading wave. For  $0.28 < H/h < 0.75$ , bores are considered partially developed and may break, while, the bore is termed fully developed and subject to large-scale turbulent breaking for  $H/h > 0.75$  (Suhayda and Pettigrew, 1977). Conceivably, Figure 1 should be modified in the very shallow water regime to account for the three bore categories.



The data for Torrey Pines and Leadbetter Beaches also depart from the  $H/h = 0.78$  slope. At Torrey Pines Beach, Thornton and Guza (1982) found waves inside the surf zone at saturation,  $H(RMS)/h = 0.44$ . A possible difference is that the  $H(RMS)$  waveheight, calculated for Torrey Pines and Leadbetter Beach data do not necessarily correspond to the individual waveheight used in the solitary wave criteria. Individual waveheight is also used for Seven Mile Beach data.

The measured phase speeds from the three experiments are compared with the four wave theories in figures 2-5. The solid line in figures 2 thru 5 denotes a perfect fit line (slope=1.0), the dashed is the 'best fit' linear equation line, and the dotted lines are the 95% confidence intervals. Table 1 is a synopsis of a linear, least squares, regression comparing measured values versus the theoretical celerity. The equations of 'best fit' lines are given as well as the correlation coefficient (Corr). All data points are unbiased (given equal weight) when computing Table 1 values.

The 95% confidence intervals (dotted lines) have been placed on Figures 2-5 for statistical purposes. Any prediction about an individual  $Y$  (actual) associated with a



given  $X$  (theory) will be most meaningful near the mean of  $X$ . Hence, the 95% confidence limits are bowed inward (toward the regression line) near the mean of  $X$ . The curvature associated with the confidence lines is slight due to the significant number (199) of data points.

A perfect fit of the data to a theory consists of a regression line whose slope is 1.0 and intercept is 0.0. The regression equation which is the best approximation of the perfect fit is presumed to be the most accurate. It is apparent from the 'best fit' (dashed) lines of figures 2-5 that hyperbolic wave theory most closely approximates the perfect solution (solid line), while linear wave theory is only slightly less accurate. Bone theory appears to give a near constant over-estimation of phase speed while solitary theory does well at the low end ( $< 500$  cm/sec) of the spectrum. Statistically, the errors tend to nullify each other when the data sets are combined. This is a recognized property of a simple linear regression and becomes quite evident as the number of data points ( $N$ ) approaches infinity.

While the equations of the least squares linear regression suggest that hyperbolic wave theory may have a slight edge; the same cannot be said if each beach is analyzed



separately. Torrey Pines Beach is best explained by bore theory, Seven Mile Beach by solitary theory, and Leadbetter Beach by hyperbolic theory. We also notice at Seven Mile Beach that none of the theories did particularly well at predicting wave speeds. This should be no surprise since 15 of the 24 data points fell outside the regime of the wave theories considered.

As can be seen from Table 1, a definitive conclusion based on the correlation coefficient would be difficult. Considering the beach totals, all four wave theories have comparable correlations; however, the regression equation for hyperbolic theory has the best slope and intercept combination, as further shown in Figure 5.

There appears to be a wide range of scatter associated with the Santa Barbara data for which there is no simple explanation. Both, Torrey Pines and Leadbetter Beach, data sets were similarly collected and analyzed. While it is true that wave refraction diagrams were required for the Santa Barbara data, there is no reason to believe that there were significant errors in the computations. The offshore sensors from Leadbetter Beach displayed little scatter (Figures 2-5) in the data; since these sensors were subject to



the largest angles of incidence as well as the largest celerities. There must be some other mechanism to explain the widespread scatter at the intermediate wave speeds.

Guza and Thornton (1982) have suggested that the presence of surf beat may be responsible for the differences between measured and theoretical celerities. Surf beat is a long reflected wave with a period of the order of several minutes, wavelength of the order of the surf zone width, and a maximum amplitude of the order of the swash at the beach face, decreasing offshore. Surf beat causes variations in the water depths, particular at the antinodes of the reflected wave, as perceived by the shorter sea-swell waves. This might explain some of the scatter of the data at Leadbetter since the phase speed is depth dependent and sensor placement could have coincided with antinodal activity which would result in a low frequency modulation of the water depth above the current meters. Surf beat also induces on-offshore velocities, particular at the nodes of the reflected waves, which can also affect the phase speed of the sea-swell waves. Sensors placed near the nodes of long waves could experience large amplitude excursions in the waves' horizontal velocities. These periodic differences in



the long wave induced velocities could result in a doppler shifting of the phase spectra and could account for scatter in Leadbetter data.



#### IV. CONCLUSIONS

Hyperbolic theory, which is an asymptotic form of cnoidal theory in shallow water, appears to give best agreement when all three data sets are collectively analyzed and compared. LeMehaute et al (1968) similarly concluded cnoidal theory gave the best comparison with laboratory data, while Dean (1965) found cnoidal theory is valid particularly in deep water but is lacking for shallow water waves. Linear theory, on the other hand, yields almost as good a comparison and is computationally easier. The data did not allow for a proper test of wave theories in the deeper water and larger waveheight regime.

Bore and solitary wave theories generally overpredicted the measured celerities while hyperbolic and linear theory tended to slightly underpredict. The differences are attributable to the relative amplitude being dispersive, as predicted by various theories. Bore and solitary theory are strongly amplitude dispersive, hyperbolic theory is weakly dispersive, and linear theory has no amplitude dispersion. Therefore, it is concluded that the waves are best categorized as weakly amplitude dispersive.



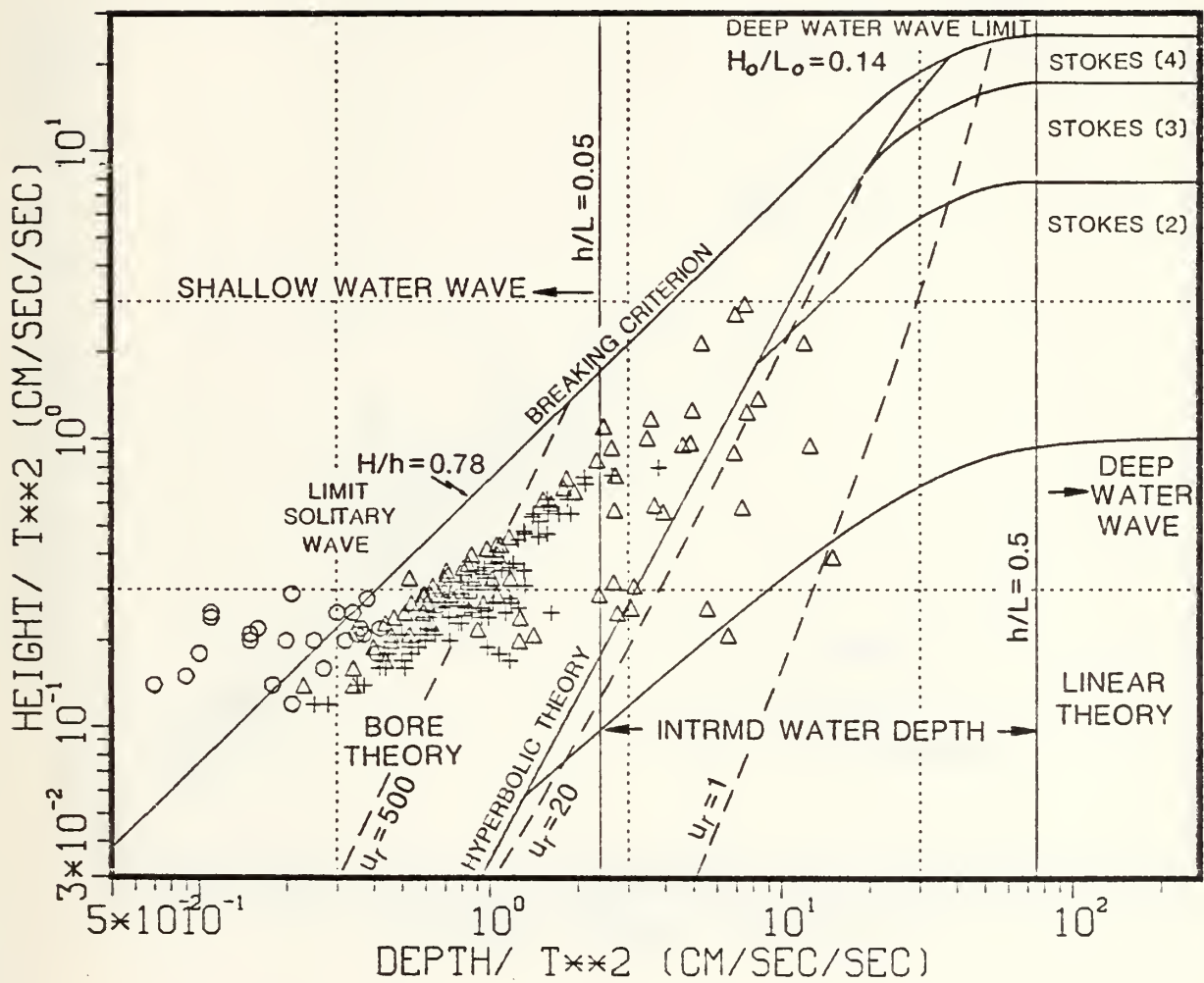


Figure 1. Regions of Applicability  
 Torrey Pines (+) Seven Mile Beach (o)  
 Leadbetter ( $\Delta$ )



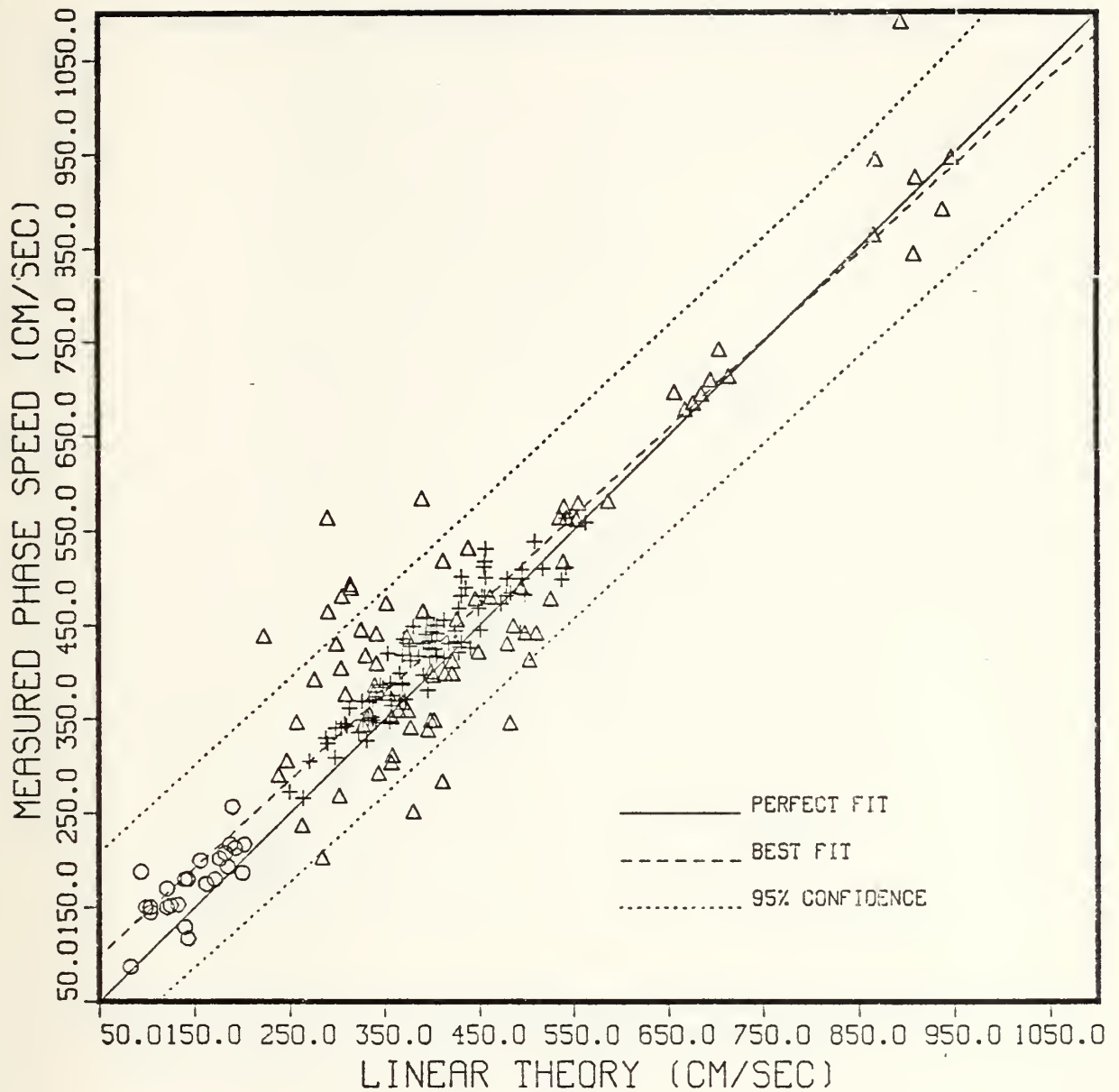


Figure 2. Measured Wave Speed vs. Linear Theory  
 Torrey Pines (+) Seven Mile Beach (o)  
 Leadbetter ( $\Delta$ )



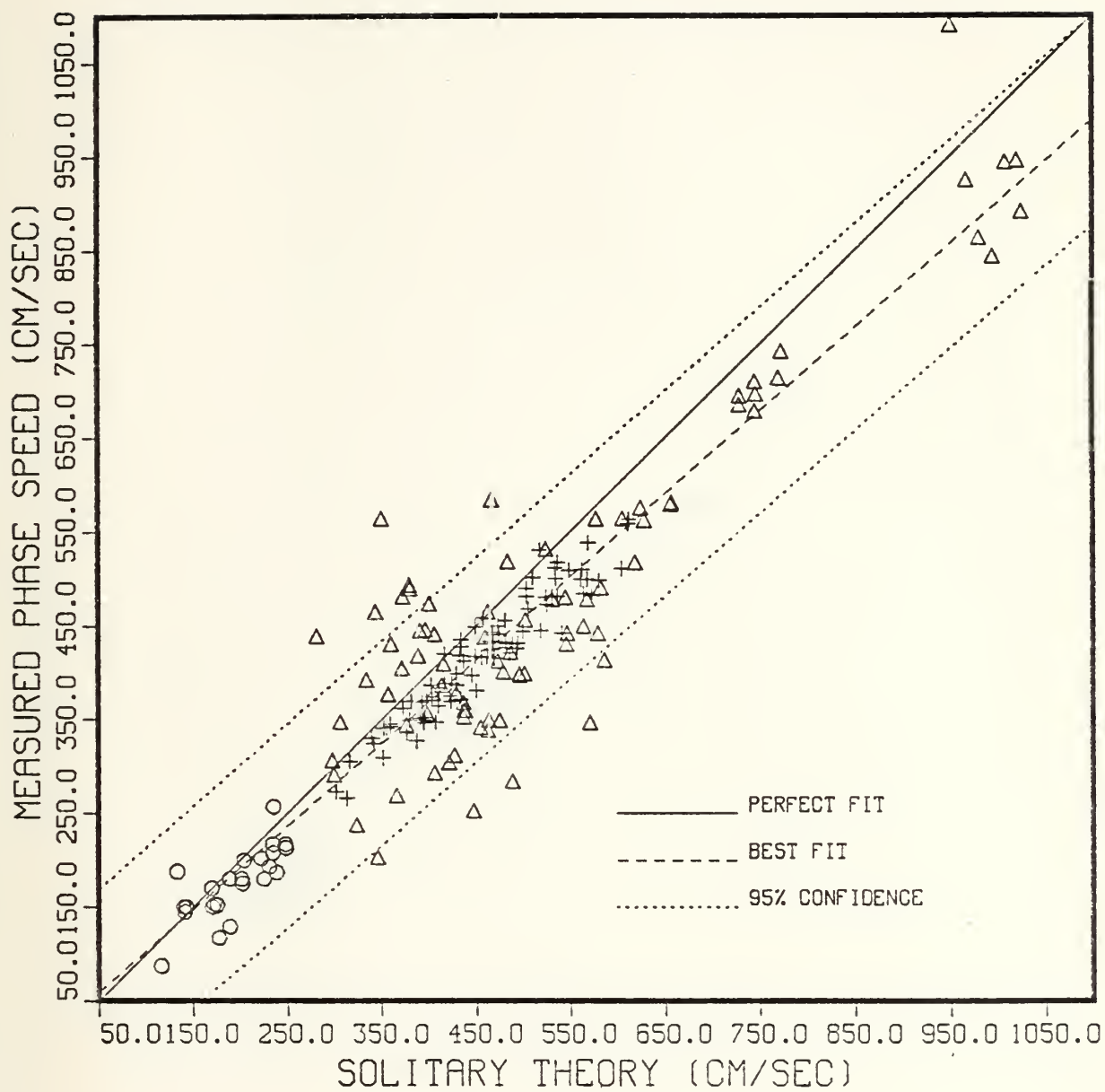


Figure 3. Measured Wave Speed vs. Solitary Theory  
 Torrey Pines (+) Seven Mile Beach (o)  
 Leadbetter ( $\Delta$ )



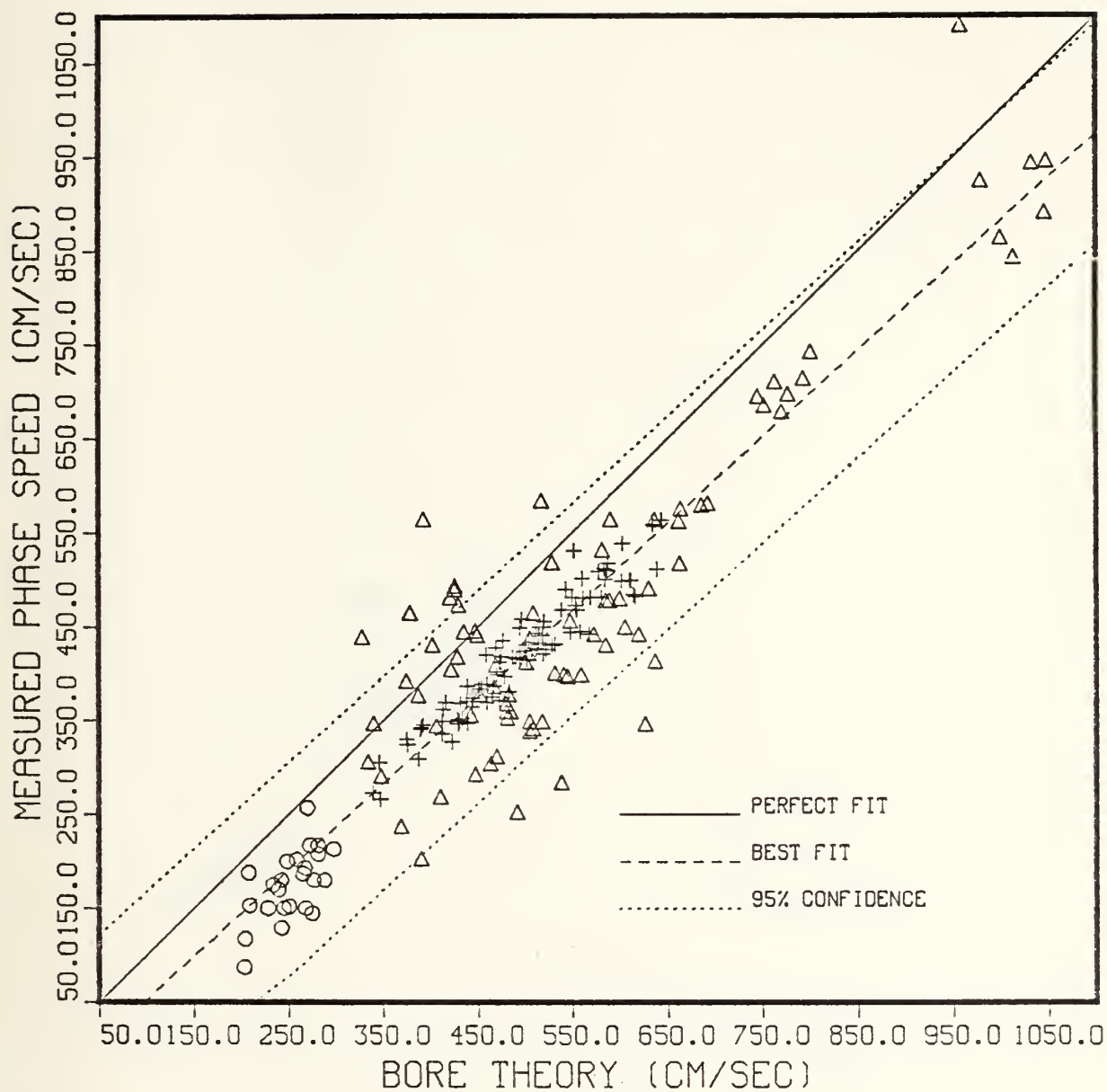


Figure 4. Measured Wave Speed vs. Bore Theory  
 Torrey Pines (+) Seven Mile Beach (o)  
 Leadbetter (Δ)



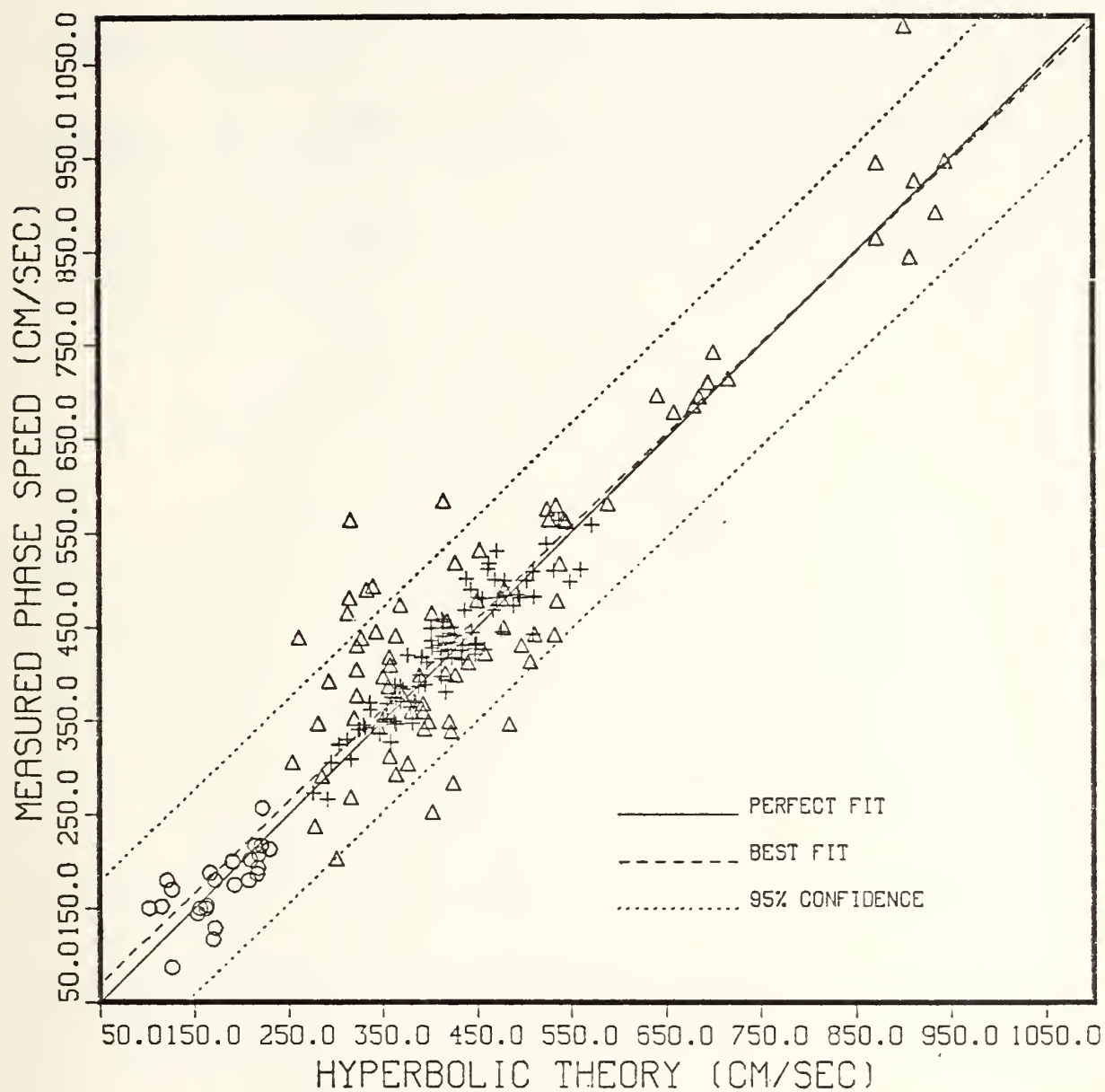


Figure 5. Measured Wave Speed vs. Hyperbolic Theory  
 Torrey Pines (+) Seven Mile Beach (o)  
 Leadbetter (Δ)



TABLE I  
Statistical Comparison of Wave Theories

|                           | <u>LINEAR</u>               | <u>SOLITARY</u>             | <u>BORE</u>                  | <u>HYPERBOLIC</u>           |
|---------------------------|-----------------------------|-----------------------------|------------------------------|-----------------------------|
| Torrey<br>Pines<br>Beach  | $Y=69.3+.884X$<br>Corr=.932 | $Y=28.6+.855X$<br>Corr=.951 | $Y=0.104+.847X$<br>Corr=.955 | $Y=39.0+.919X$<br>Corr=.920 |
| Seven<br>Mile<br>Beach    | $Y=62.5+.759X$<br>Corr=.749 | $Y=36.3+.717X$<br>Corr=.768 | $Y=-38.0+.844X$<br>Corr=.625 | $Y=57.0+.667X$<br>Corr=.697 |
| Lead-<br>better<br>Beach  | $Y=74.5+.896X$<br>Corr=.897 | $Y=30.6+.862X$<br>Corr=.891 | $Y=-18.9+.890X$<br>Corr=.883 | $Y=51.2+.930X$<br>Corr=.900 |
| Total<br>of all<br>Beachs | $Y=51.9+.932X$<br>Corr=.937 | $Y=14.5+.886X$<br>Corr=.936 | $Y=-39.6+.922X$<br>Corr=.931 | $Y=20.2+.975X$<br>Corr=.936 |



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